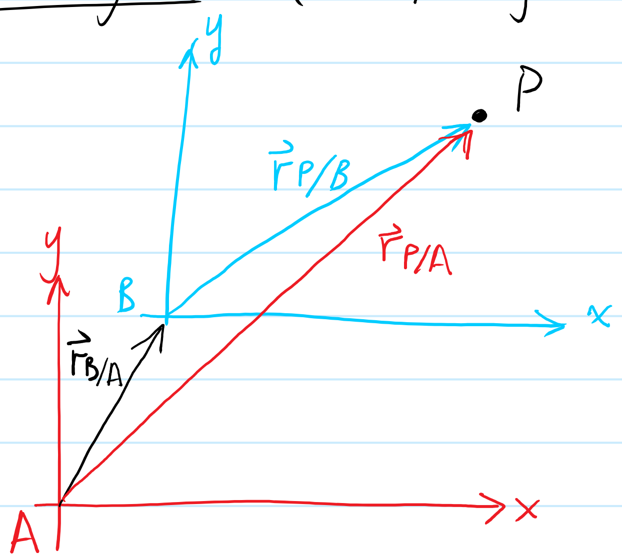
Lecture 2 : Newton's Law I.

Outline:
- Relative motion, Reference frame
- Newton's Laws
- Inertial frame
- Apparent weight

Reference frame (choice of coordinate system)



$\vec{r}_{P/A}$: position of P observed by / relative to A

$\vec{r}_{P/B}$: position of P

According to vector addition rule: $\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A}$. ← contract.

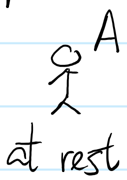
or $\vec{r}_{P/B} = \underbrace{\vec{r}_{P/A} - \vec{r}_{B/A}}_{\text{relative to same observer}}$

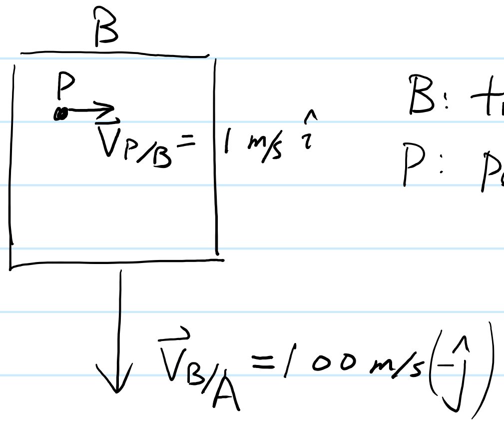
Taking time derivatives:

$$\vec{V}_{P/A} = \vec{V}_{P/B} + \vec{V}_{B/A} \quad \text{or} \quad \vec{V}_{P/B} = \vec{V}_{P/A} - \vec{V}_{B/A}$$

$$\vec{a}_{P/A} = \vec{a}_{P/B} + \vec{a}_{B/A} \quad \text{or} \quad \vec{a}_{P/B} = \vec{a}_{P/A} - \vec{a}_{B/A}.$$

Example.



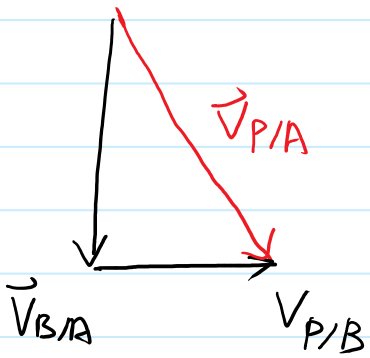


B: train

P: Person in the train

velocity of P observed by A:

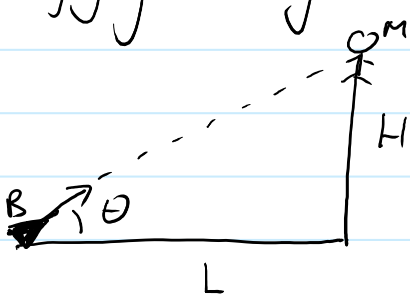
$$\vec{V}_{P/A} = \vec{V}_{P/B} + \vec{V}_{B/A} = -100\hat{j} + 1\hat{i} = \hat{i} + 100\hat{j}.$$



Review on monkey & hunter.

at $t=0$, monkey sees the bullet flying directly towards it.

$$\because \quad \tan\theta = \frac{H}{L}$$



What is the trajectory of the bullet in the monkey's point of view afterwards?

$\Rightarrow$ what is $\vec{V}_{B/M}(t)$?

According to Lecture 1, the trajectories are found relative to a stationary observer, say the ground (G).

$$\vec{V}_{B/G} = V_0 \cos\theta \, \hat{i} + (V_0 \sin\theta - gt)\hat{j}$$

$$\vec{V}_{M/G} = 0 \, \hat{i} + (-gt)\hat{j} \quad \text{(velocity for free fall)}$$

$$\Rightarrow \vec{V}_{B/M} = \vec{V}_{B/G} - \vec{V}_{M/G}$$

$$= V_0 \cos\theta \, \hat{i} + V_0 \sin\theta \, \hat{j}$$

$$= \vec{V}_0 \quad \text{(constant)}$$

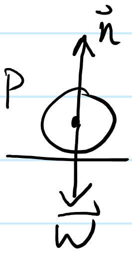
$\Rightarrow$ The falling monkey sees the bullet is flying in a straight line towards it all the time.

<u>Inertia</u> <u>1st Law</u>

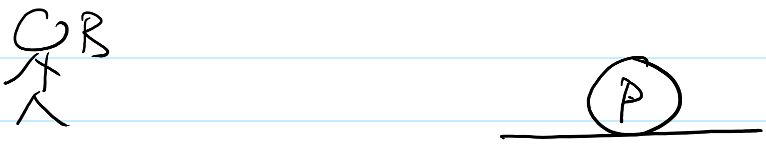
" $\vec{F}_{net} = \Sigma \vec{F}_i = \vec{0} \iff$ uniform motion, $\vec{a} = \vec{0}$ "

1st Law is not true for all reference frame, why?

Consider a ball being at rest on the floor.

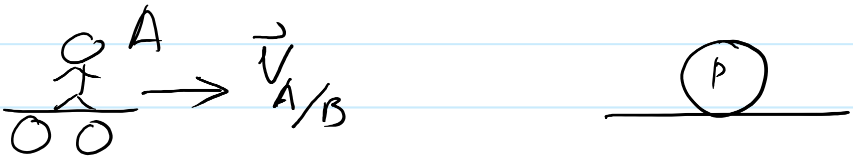


$W = n \Rightarrow \vec{F}_{net} = \vec{0} \xrightarrow{\text{1st Law}} \vec{a}_p = \vec{0}$.



at rest

$$\vec{a}_{P/B} = \vec{0} \qquad \checkmark \quad \text{1st Law ok!}$$



$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} = \vec{v}_{P/B} - \vec{v}_{A/B}$$

$$\Rightarrow \quad \vec{a}_{P/A} = \vec{a}_{P/B}^{\;\nearrow 0} - \vec{a}_{A/B}$$

Case (a) : A is moving in uniform motion relative to B.

$$\Rightarrow \vec{a}_{A/B} = \vec{0}$$

$$\Rightarrow \quad \vec{a}_{P/A} = \vec{0} \qquad \checkmark \quad \text{1st Law ok!}$$

Case (b) : A has none zero acc. relative to B.

$$\Rightarrow \vec{a}_{A/B} \neq \vec{0}$$

$$\Rightarrow \quad \vec{a}_{P/A} \neq \vec{0} \quad \text{but} \quad \vec{F}_{net} \text{ on } P = \vec{0}$$

$\Rightarrow$ 1st Law is violated.

Let's define the referance frame where Newton's 1st Law is valid as an <u>inertial frame</u>.

Newton's 1st Law (& 2^{nd} Law) does not hold in a reference frame (A in case (b)) which has acc. relative to an inertial frame (B).

We call those frame a <u>non-inertial frame</u>.

$$\boxed{\text{Newton's 1st and 2nd Law apply only in inertial frame}}$$

How to find inertial frame?

- an observer with no net forces acting on it is an inertial frame.
- an observer moves in uniform motion relative to an inertial observer.

Newton's 2nd Law.

$$\vec{F}_{net} = m\vec{a} \Rightarrow |\vec{a}| = \frac{|\vec{F}|}{m}$$

$|\vec{F}|$ ← cause

$|\vec{a}|$ ← effect

m ← mass "laziness"

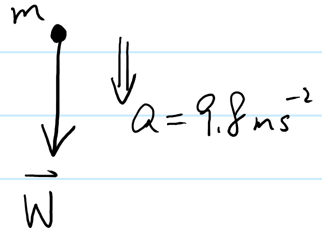
SI unit: $1 \text{ Newton} = 1 \text{ N} = 1 \text{ kg m s}^{-2}$

Why is weight $= mg$?

Weight = gravitation attraction due to the Earth.

On surface of the Earth, we observe that every free falling object falls at $a = 9.8 \text{ m s}^{-2}$.

Since no forces beside gravity is acting on the object, according to 2nd Law.



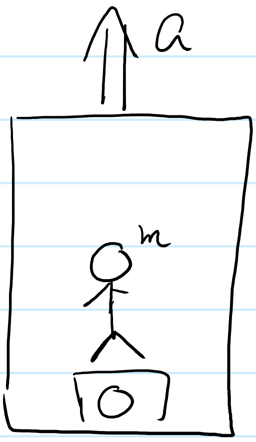
$$F_{net} = m\,a$$

$$\downarrow \qquad \downarrow \text{ from kinematics.}$$

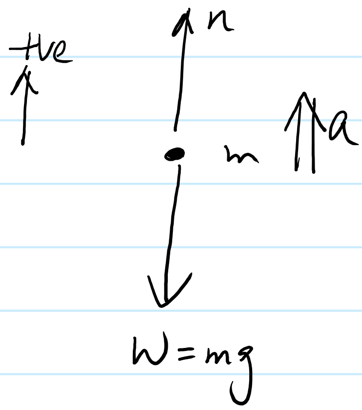
$$W = m\,g$$

Application

Consider



Free Body Diagram



$$F = ma$$ acc of the lift and the person

$$\underbrace{n - W}_{\text{causes}} = m \underbrace{a}_{\text{effect}}$$

$$\Rightarrow \quad n - mg = ma$$ acceleration relative to a stationary observer. A

$$n = m(a+g) > mg = W$$

reading on the scale

Reading is larger than actual weight.

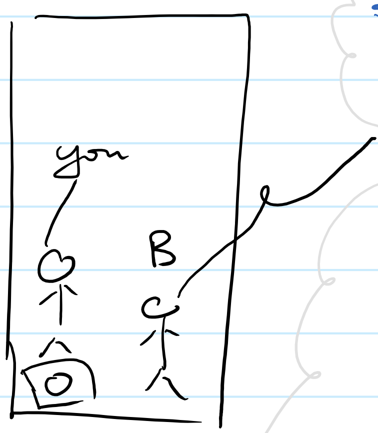
Apparent weight = mass × relative acc. to some non-inertial frame (e.g. the lift)

Suppose $m = 50\ kg$, $g = 10\ ms^{-2}$ and $a = 1\ ms^{-2}$

$\Rightarrow n = 50 \cdot 11 = 550\ N$

$\Rightarrow$ Reading on the scale $\sim 55\ kg$

Consider someone (B) inside the lift and he/she wants to apply Newton's 2^{nd} Law on you.



In B's mind

- There are gravity and normal force acting on this guy. (you)

$\Rightarrow F_{net} = n - W$

- But this guy is at rest.

Newton's Law says:

$$\Rightarrow \vec{a} = \vec{0}$$

$$F_{net} = ma$$

$$n - W = m \cdot 0$$

$$\Rightarrow n = W = mg$$

not $m(g+a)$ in last calculation.

Since the reading on the scale is $55\ kg$, he must be $55\ kg$.

But you are only $50\ kg$. What's wrong?

Newton's 3rd Law: Action-Reaction pair:

If body A exerts a force on body B (an "action"), then body B exerts a force on body A (a "reaction"). These two forces have the **same magnitude** but are **opposite** in direction. These two forces act on different bodies.